A Method to Place Meters in Active Low Voltage Distribution Networks using BPSO Algorithm

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Abstract—This paper proposes a method to be used by a DSO to optimally place sensors at MV/LV substation and some low voltage cable distribution cabinets. This method aims to improve the estimation of the grid states at low voltage distribution networks. This method formulates a multi-objective optimization problem to determine the optimal meter placement configuration. This formulation minimizes the low voltage state estimation error and the cost associated to a particular meter deployment configuration. The method uses Binary Particle Swarm Optimization (BPSO) to solve the optimization problem and it has been tested on a network based on the Cigré LV benchmark grid. The simulation results show that the method can be applied to both situations where smart meter measurements are available and situations where they are not. In the latter situation the measurements are replaced by pseudo-measurements, which represent meter readings by using smart meter historical data and prediction models.

Keywords—Active Low Voltage Distribution Networks, BPSO, Meter Placement Problem, Multi-objective Optimization, Observability Analysis, State Estimation.

I. INTRODUCTION

The integration of distributed energy resources (RES) at radial low voltage distribution networks is becoming more popular. Typical examples are photovoltaic power stations mounted on the rooftop of residential or commercial buildings. The massive installations of these systems can induce power quality problems in the grid (e.g. reverse power flows, increase in power losses and overvoltages) [1]. Current power system regulation models can specify that part of Distribution System Operators’ (DSO) revenue has to be based on how well they cope with their quality targets [2]. Therefore, DSOs are incentivized to invest in enhancing the monitoring and the supervision of the low voltage side of the electrical grid. Since nowadays this part of the grid is poorly observed (e.g. very few MV/LV substations are monitored), this work focuses on improving the estimation of the grid states at this level. Hence, it is important to determine where in the grid the DSOs should place meters to be able to perform state estimation using the required minimum number of meters.

There are relevant publications that have addressed the meter placement problem in active distribution networks e.g. [3] proposes a heuristic approach to identify the measuring points on the network with higher voltage variations, [4] and [5] present the meter placement as a stochastic optimization problem using Monte Carlo simulations, and in the dissertation presented in [6], the author provides a thorough and updated literature review on power distribution system state estimation and meter placement algorithms. In the latter work special emphasis is put on both distributed state estimation as a future challenge and on the unbalanced characteristics of the lines.

In our study, balanced three phase lines are considered due to that being the case in the Swedish LV distribution system. Additionally, the backward/forward sweep method is applied in section II.E to run the load flow calculation in the branch current state estimation. This restricts the network topology to radial networks but this is mostly the case in LV distribution.

This paper is organized as follows: First, the meter placement problem is introduced and then the proposed method is presented. After that, a case-study is described and finally the conclusions and the perspectives for future research work are introduced.

II. METHOD

A. Meter Placement Problem Description

The multi-objective problem presented in this paper consists of determining the optimal meter placement configuration in distribution LV networks. The optimality is determined by the low voltage state estimation error and the cost associated to a particular meter deployment configuration. Two main cases are studied and described in TABLE 1.

TABLE 1. CASE-STUDIES.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Characteristics that define each case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>- Smart meter measurements are not available and instead pseudo-measurements are used.</td>
</tr>
<tr>
<td>Case II</td>
<td>- MV/LV substation is monitored (V and I).</td>
</tr>
<tr>
<td>Case II</td>
<td>- Degree of freedom: The required branches at certain LV cable distribution cabinets are monitored (V and I).</td>
</tr>
</tbody>
</table>

This work is supported by the SweGRIDS programme and the European Union Seventh Framework Programme (FP7/2007-2013) under Grant Agreement no 308913.
Case I describes a situation where smart meter measurements are not available. This can be the case for countries where the DSO is only allowed to use the meter readings for billing purposes but not for system operation activities. Instead, pseudo-measurements based on consumer’s historical data and statistical load models can be used to create a representation of meter readings: the resultant of combining load consumption and PV generation. Case II instead considers that the DSO is able and allowed to use the smart meter measurements for system operation activities (e.g., LV monitoring) apart from for billing activities.

The MV/LV substation is monitored for both Case I and Case II. This means two things: first, the low voltage side of the transformer is measured with a voltage sensor. And second, all the feeders connected to that feeder bus are measured with current sensors.

In both Case I and Case II, the degree of freedom that permits obtaining an optimal solution corresponds to monitoring the required branches at certain LV cable distribution cabinets. This means that for each specific network topology configuration, the fact of monitoring some branches at certain LV cable distribution cabinets can improve the quality of the state estimation that much that it would pay off at certain LV cable distribution cabinets. This can be the case for the meter installation cost. The schematic diagram in Figure 1 shows the location of the LV cable distribution cabinets (LVCDC) where the monitoring system can be deployed.

![Schematic diagram showing the LVCDC](image)

**Figure 1. LV Schematic diagram showing the LVCDC.**

### B. Mathematical Problem Formulation

The multi-objective optimization problem, see equation (1), consists of minimizing the weighted Euclidean norm of the vector formed by two components: the first component reflects the cost of meter configuration (CM) and the second component reflects the accuracy of the estimation error defined by the voltage error estimation (VEE). The components are normalized with respect to the maximum values \(V_{\text{EE}(\hat{X})_{\text{max}}}\) and \(C_{\text{M}(\hat{X})_{\text{max}}}\) so that both components can be in the same scale: from 0 to 1. The weights \(w_{fe}\) and \(w_{fc}\) correspond to the voltage error and cost weight factors that can be used to prioritize one component against the other if required.

\[
\begin{align*}
\min F(\hat{X}) &= \| w_{fe} \cdot V_{\text{EE}(\hat{X})} + w_{fc} \cdot C_{\text{M}(\hat{X})} \|_2 \\
&= \sqrt{(w_{fe} \cdot V_{\text{EE}(\hat{X})})_{\text{max}}^2 + (w_{fc} \cdot C_{\text{M}(\hat{X})})_{\text{max}}^2}
\end{align*}
\]

(1)

The CM component is defined in equation (2) and it quantifies the monetary cost of a particular meter configuration within a feeder. It reflects the unit cost associated to voltage sensors, current sensors and smart meters.

\[
CM = (N_{Vs} \cdot UC_{Vs}) + (N_{Ic} \cdot UC_{Ic}) + (N_{SM} \cdot UC_{SM})
\]

(2)

Where,
- \(N_{Vs}\): number of voltage sensors.
- \(N_{Ic}\): number of current sensors.
- \(N_{SM}\): number of smart meters.
- \(UC_{Vs}\): unit cost of voltage sensors.
- \(UC_{Ic}\): unit cost of current sensors.
- \(UC_{SM}\): unit cost of smart meters.

The VEE component is defined as the summation of the square roots of the difference squared between the real and imaginary parts of the reference voltage vector and the estimated voltage vector, and it is normalized to the magnitude of the reference voltage vector. See equation (3).

\[
VEE = \sum_{i=1}^{N_T} \sqrt{(V_{\text{rest}} - V_{\text{ir}})^2 + (V_{\text{iref}} - V_{\text{ir}})^2}
\]

(3)

Where,
- \(V_{\text{rest}}\): real part of the estimated voltage vector.
- \(V_{\text{iref}}\): imaginary part of the estimated voltage vector.
- \(V_{\text{ir}}\): real part of the reference voltage vector.
- \(V_{\text{ir}}\): imaginary part of the reference voltage vector.
- \(N_T\): total number of buses in the network (\(N_T = N_v + N_c\)).

Both the CM and VEE values are obtained for a specific decision variable vector configuration: \(\hat{X}\), which consists of six binary vectors as defined in equation (4) and described in

\[
\hat{X} = [X_v, X_{I_{bus}}, X_{I_{branchc}}, X_{I_{branchr}}, X_{SM_{bus}}, X_{SM_{bus}}]
\]

(4)

**Table 2. Description of the Decision Variable Vector \(\hat{X}\).**

<table>
<thead>
<tr>
<th>Elements of (\hat{X})</th>
<th>Size of vector</th>
<th>Description of the vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_v)</td>
<td>(1 \times N_v)</td>
<td>Vector that specifies the voltage sensor location at LV cable distribution cabinets. A value of 1 in a position of the vector represents a voltage sensor at the LV cable distribution cabinet associated to that position.</td>
</tr>
<tr>
<td>(X_{I_{bus}})</td>
<td>(1 \times N_v)</td>
<td>Vector that specifies the zero injection buses. A value of 1 in a position of the vector represents a zero injection bus at the LV cable distribution cabinet associated to that position.</td>
</tr>
<tr>
<td>(X_{I_{branchc}}) (from)</td>
<td>(1 \times N_L)</td>
<td>Vector that together with (I_{branchc}) is used to specify the monitored branches at LV cable distribution cabinets.</td>
</tr>
<tr>
<td>(X_{I_{branchr}}) (to)</td>
<td>(1 \times N_L)</td>
<td>Vector that together with (I_{branchr}) is used to specify the monitored branches at LV cable distribution cabinets.</td>
</tr>
<tr>
<td>(X_{SM_{bus}})</td>
<td>(1 \times N_B)</td>
<td>Vector that specifies the smart meter location at LV networks. A value of 1 in a position of the vector represents a SM at the bus associated to that position.</td>
</tr>
<tr>
<td>(X_{SM_{bus}})</td>
<td>(1 \times N_B)</td>
<td>Vector that specifies the pseudo meter location at LV networks. A value of 1 in a position of the vector represents a PS at the bus associated to that position.</td>
</tr>
</tbody>
</table>
\(N_c\) represents the total number of LV cable distribution cabinets in the LV feeder, where current and voltage sensors can be deployed. \(N_b\) represents the total number of branches connected to the LV cable distribution cabinets. And \(N_h\) represents the total number of households in the LV feeder, where either SM or pseudo-measurements are used.

Consequently, the size of the \(\tilde{X}\) vector is determined as depicted in equation (5).

\[
1 \times (2 \cdot N_c + 2 \cdot N_b + 2 \cdot N_h)
\]

The constraints of the optimization problem determine that the observability of the LV system should be achieved and that the \(I_{bus}, SM_{bus}\) and the \(PS_{bus}\) vectors should be specified based on the topology of the network. This means that the end node positions (e.g. households) and zero injection buses shall be determined a priori before the optimization problem is solved. Additionally, \(SM_{bus}\) and \(PS_{bus}\) are complementary vectors. In this setup either pseudo-measurements or smart meter measurements are applied, but not both at the same time.

C. Measurement alternatives

To address the meter placement problem, it turns necessary to specify the type of available instrumentation and its characteristics. The following sub-sections will explain the specifics.

1) Meter Measurements

In this study, the MV/LV secondary substations are assumed to be monitored by voltage and current sensors. The meter measurements specifically refer to the instrumentation used for monitoring the phase voltages and the required branch currents at certain LV cable distribution cabinets, in such a way that the active and reactive power flows can be derived.

2) Smart Meters (SM)

Currently, typical smart meters have the capability to collect many electrical parameters (e.g. power and energy registers, tariff registers, voltage and current registers, etc.). In this study, the end-point nodes are considered as PQ nodes and consequently, the SM’s active and reactive power registers are used, which represent import (consumption) or export (generation) power: \(P_{imp}\) \(Q_{imp}\).

3) Pseudo-measurements

There are situations where smart meter measurements are not available (e.g. \(P_{imp}\), \(Q_{imp}\)) and instead the so-called pseudo-measurements are applied. This type of measurements relies on different type of consumer load modeling, based on statistical processing of smart meter historical data from similar systems to provide measurement estimations to unmeasured loads [7]. These models provide an average expected value with a certain accuracy, which increases as the elements under estimation are aggregated. For example, it is more accurate to estimate the consumption at the substation level than at household level. Furthermore, the time aspect also has a significant effect on the quality of the standard deviation of the estimated pseudo-measurement. As explained in [8], the estimation effectiveness decreases as the time delays in data transmission rise.

D. Proposed Method Using BPSO Algorithm

The proposed method is based on the discrete binary version of the general metaheuristic approach known as Particle Swarm Optimization (PSO) algorithm. Proper description and explanations for binary PSO (BPSO) can be found in [9] and [10]. As a summary, the main characteristics of BPSO algorithm are provided as follows:

- Each \(i\)th particle corresponds to an instance of the decision variable \(\tilde{X}\), i.e. \(\tilde{X}_i = [x_1^i, ..., x_k^i, ..., x_{dp}^i]\), where \(k\) represents the iteration number and \(dp\) is the dimension of the particle.
- The population is a set of particles in the swarm, where \(N_{pp}\) represents the number of particle in the population, i.e. \(POP = [\tilde{X}_1, ..., \tilde{X}_i, ..., \tilde{X}_n_{pp}]\).
- The best position of particle \(i\) at iteration \(k\) represents the position whose fitness value is the lowest among the previous particle’s position, i.e. \(PB^k_i = [p_b^k_i, ..., p_b^{k-1}_i, ..., p_b^1_i]\).
- The best \(i\)th particle among all the best particles (\(PB^k\)) at the current iteration \(k\) represents the global best particle, i.e. \(GB^k = [g_b^k, ..., g_b^{k-1}, ..., g_b^1]\).
- At each \(k\)th iteration each \(i\) particle is associated to a position \((\tilde{X}_i^k)\) and also to a velocity, i.e. \(V_i^k = [v_1^k, ..., v_{dp}^k]\). The velocity vector is updated at each \(k\)th iteration by equation (7). Where \(w\) is the inertia weight, \(c_1\) and \(c_2\) are the acceleration constants and \(r_1\) and \(r_2\) are two uniform random numbers in the range \([0, 1]\).

\[
V_i^{k+1} = wV_i^k + c_1r_1(\text{p_b}^k - X_i^k) + c_2r_2(\text{g_b}^k - X_i^k)
\]

\(w = w_2 + (w_1 - w_2) \cdot \left(\frac{k_{max} - k}{k_{max}}\right)\)

The \(w\) inertia weight is calculated as defined in equation (8), where \(w_1\) and \(w_2\) are respectively the initial value and the final value for the inertia weight, \(k_{max}\) represents the maximum number of iterations and \(k\) the current iteration.
Similarly, the \( r \)th particle needs to be updated at each \( k \)th iteration by equation (9). However, to apply the binary particular case, which specifies that the PSO should find the solution in a binary search space, a sigmoid transformation to the velocity component is applied as specified in equation (10), and therefore, equation (11) replaces equation (9).

\[
X_{i}^{k+1} = X_{i}^{k} + \psi_{i}^{k+1}
\]

\[
Sigmoid(\psi_{i}^{k+1}) = \frac{1}{1 + e^{-\psi_{i}^{k+1}}}
\]

\[
X_{i}^{k+1} = \begin{cases} 
1, & r < Sigmoid(\psi_{i}^{k+1}) \\
0, & \text{otherwise}
\end{cases}
\]

Where \( r \) is a uniform random number in the range \([0, 1] \).

The proposed method considers the previously detailed BPSO characteristics (see Figure 2) and described in the following steps:

- **Step 0:** Initialization. At this step the initial particle swarm is randomly generated and the position and velocities of the particles are initialized, i.e. \( X_{i}^{1} = [x_{1}^{1}, ..., x_{nP}^{1}] \) and \( \bar{V}_{i}^{1} = [v_{1}^{1}, ..., v_{nP}^{1}] \). In this initial swarm, the two extreme situations are also considered: the configuration without any additional measurement devices (minimal cost and maximum state estimation error) and the configuration where the measurement devices are installed on each node and branch (high cost and minimum state estimation error). Then, the initial configuration is assessed and the corresponding CM and VEE values are calculated. After that, the initial fitness function \( FF(X_{i}^{1}) \) is calculated using equation (12) and the initial particle best \( (PB)^{1} \) and global best \( (GB)^{1} \) are found.

\[
FF(X_{i}^{k}) \equiv \sqrt{w_{V} \cdot \frac{VEE(X_{i}^{k})}{VEE_{max}}} + \left( w_{C} \cdot \frac{CM(X_{i}^{k})}{CM_{max}} \right)^{2}
\]

- **Step 1:** Evaluate the termination criteria. At this step the algorithm is stopped if any of the following optimality conditions apply:
  - The number of iterations exceeds the maximum number of iterations: \( k_{max} > k \).
  - The fitness function of the GB particle remains stable for a number of iterations \( k_{breakloop} \).
- **Step 2:** Update the counter \( (k = k + 1) \).
- **Step 3:** Calculate the new velocity \( (\bar{V}_{i}^{k}) \) and new position of the swarm \( (X_{i}^{k}) \). With the positions a new swarm configuration is obtained and therefore new meter placement alternatives.
- **Step 4:** The new configuration is assessed and the corresponding CM and VEE values are calculated.

- **Step 5:** Evaluate the fitness function \( (FF(X_{i}^{k})) \) and find particle best PB \( (PB)^{k} \) and GB \( (GB)^{k} \) and global best using equation (13) and (14).

\[
\begin{align*}
PB_{i}^{k} & = X_{i}^{k}, & FF(X_{i}^{k}) < FF(PB_{i}^{k-1}) \\
PB_{i}^{k} & = PB_{i}^{k-1}, & \text{otherwise}
\end{align*}
\]

\[
\begin{align*}
GB^{k} & = GB^{k}, & \min\{FF(PB^{k})\} < FF(GB^{k-1}) \\
GB^{k} & = GB^{k-1}, & \text{otherwise}
\end{align*}
\]

E. Meter Placement Configuration Assessment

The meter placement configuration assessment part of the algorithm is run to be able to obtain the CM and VEE components for each particle (i.e., meter configuration). It requires two inputs: the branch matrix representing the LV distribution network and the meter placement configuration for the LV network under study. The Figure 3 shows the block diagram description.
The observability analysis is executed to check whether the meter deployment for a specified configuration is adequate or not to meet the constraint of full observability of the network, it also checks that the measurements are sufficient to perform state estimation. Since the network under study is a radial LV distribution system, the non-observable situations that could arise if a meter would break down are not that critical as they would be at higher distribution levels or at the transmission grid. Therefore, there is no necessity for the grid observability to contain redundancies if that implies deploying additional devices. This is the reason why a redundancy check is performed in addition to the observability test, as presented in [11], where authors propose a numerical approach for observability analysis considering redundant measurements.

2) CM Calculation
Starting from the LV distribution network and assuming full observability, the load flow study is performed to obtain all the variables of interest for a given configuration (i.e. complex voltages for all nodes, complex currents and power flows along all branches of the network). The results from the load flow study are considered to be the reference situation and these results are compared to the results obtained from the state estimation, so that the estimation error can be calculated.

The set of measurements is generated using a meter deployment configuration that fulfills the observability criteria. The measurement values are obtained by adding a normally distributed error to the reference situation. The standard deviation (σ) of the error depends on the measurement alternative used (i.e. meter or pseudo-meter). Once the set of meters that form a particular configuration are known the CM is calculated as defined in equation (2). The CM parameters covers the investment costs (e.g. Capital Expenditures (CAPEX) [12]) and the Operational Expenditures (OPEX) [13].

Similarly, the net present value analysis (NPV) [14] can be applied for each asset deployment configuration considering e.g. the next 10 years after the deployment. The NPV in a financial tool that can be used for helping in decision making processes such as in expansion projects like this. It takes into account the asset investment (e.g. meters), the depreciation of the assets and the cash flow. The meter deployment configuration with a positive NPV is a profitable investment while negative NPV corresponds to financial losses. Therefore, NPV maximization could be used to replace CM indicator minimization in the equation (1).

3) Branch Current State Estimation
Each deployment configuration provides a set of measurements that unfortunately, does not contain the complete state of the system. However, there are techniques such as state estimation, which take all the measurements and use them to determine the principal behavior of the system at any point in time. The SE technique is largely applied at the transmission network, being the nodes’ complex voltages the states variables. In distribution networks however, due to the radial nature of the networks, it is preferable to use the branch complex currents as state variables because the computation is faster, more robust and the current measurements are easier to handle [15]-[16]. The Weighted Least Square (WLS) estimator is applied and it is based on the minimization of weighted measurement residuals. It is formulated as equation (15).

\[
\text{Min. } J(x) = [z - h(x)]^T R^{-1} [z - h(x)]
\] (15)

Where, 
\(x\): system state vector.
\(z_i\) = \(h_i(x) + e_i\): \(i\)th measurements.
\(h_i(x)\): \(i\)th nonlinear function relating meas. \(i\) to the state variable.
\(e_i\): \(i\)th measurement error.

Once the complex currents are obtained, it is possible to take advantage of the radial characteristic of the network and calculate the nodes’ complex voltages by distribution load flow using the backward/forward sweep method [17]. Balanced three-phase lines are assumed (e.g. Swedish case) and this allows to apply the three phase to single phase equivalence transformation.

4) VEE Calculation
The VEE indicator as defined in equation (3) can be calculated by using the complex voltages from the reference situation and the branch current SE.

The SE results can be biased by the randomness in the error function that is used to create the set of measurements. Therefore, a way to avoid such bias effect is to apply Monte Carlo method for \(N_{iter}\) iterations each time using a different set of measurements generated through the error probability function. And the resulting VEE average value is used.

III. CASE-STUDY

A. Case-Study Description
The proposed meter deployment method is illustrated with a case-study and four different scenarios are applied to the feeder shown in Figure 4. This feeder corresponds to a residential feeder, which contains 12 active residential nodes (e.g. prosumers), 22 branches and 10 LVDCDC. The cable characteristics are obtained from the network proposed in [18].

The four different scenarios are simulated in this study as depicted in TABLE 3. These simulated scenarios are based on Case I (i.e. pseudo-measurements) and Case II (i.e. smart meters) with variations in the weight factors.
are represented by a "+" sign and two identification numbers. LVCDC (1+1 particle). The different configuration solutions representing the extreme configurations, i.e. meters deployed in all the LVCDC (1+2 particle) and no meter deployed in any LVCDC (1+1 particle). The different configuration solutions are represented by a "+" sign and two identification numbers. The number on the side represents the iteration number and the upper right number identifies the particle of the population. In scenarios #1 and #2 a horizontal offset can be observed that represents the cost of having physical SM meters. The configuration solutions that during the optimization cycle have been generated more than once are identified with a magenta color and the solutions that have been considered as global best are identified with the green "*" sign. A Pareto front can be observed in all the simulated scenarios. This front is formed by all the equally optimal solutions. In this work a single optimal solution is obtained by minimizing the Euclidean norm of the vector that defines each configuration solution. This single optimal solution is identified by a red circle, which represents the final global best.

In Figure 6 the evolution of the fitness function and the components forming the fitness function are shown. The algorithm finds the optimal solution after 10 iterations for scenarios characterized by same weight factors and it converges faster for the scenarios that prioritize the configuration cost (e.g. scenario #2 and #4). In all the simulated scenarios the algorithm stops when one of the optimality conditions is met, i.e. the fitness function of the GB particle remains stable for 10 iterations.

<table>
<thead>
<tr>
<th>Scenario #</th>
<th>Case I / Case II</th>
<th>w_fe</th>
<th>w_SM</th>
<th>σ_SM</th>
<th>σ_PS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Case II</td>
<td>1</td>
<td>1</td>
<td>0.0001</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Case II</td>
<td>1</td>
<td>9</td>
<td>0.0001</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Case I</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>0.003</td>
</tr>
<tr>
<td>4</td>
<td>Case I</td>
<td>1</td>
<td>9</td>
<td>-</td>
<td>0.003</td>
</tr>
</tbody>
</table>

The BPSO parameters are specified in Table 4.

TABLE 4. BPSO SIMULATION PARAMETERS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_pp</td>
<td>30</td>
</tr>
<tr>
<td>N_iter</td>
<td>200</td>
</tr>
<tr>
<td>c_1</td>
<td>0.5</td>
</tr>
<tr>
<td>c_2</td>
<td>0.3</td>
</tr>
<tr>
<td>w_1</td>
<td>0.3</td>
</tr>
<tr>
<td>w_2</td>
<td>0.1</td>
</tr>
<tr>
<td>k_max</td>
<td>200</td>
</tr>
<tr>
<td>k_breakloop</td>
<td>10</td>
</tr>
</tbody>
</table>

B. Simulation Results

In Figure 5 the simulations results are shown for the four scenarios. X-axis represents the cost of meter configuration (CM) and Y-axis represents the voltage error estimation (VEE). Both axes are normalized to the maximum values representing the extreme configurations, i.e. meters deployed in all the LVCDC (1+2 particle) and no meter deployed in any LVCDC (1+1 particle). The different configuration solutions are represented by a "+" sign and two identification numbers. The number on the side represents the iteration number and the

![Figure 4. Modified Cigré LV Benchmark network](image)

![Figure 5. The simulation results showing the Pareto front for each simulated scenario. From left to right: scenario #1, scenario #2, scenario #3 and scenario #4.](image)

![Figure 6. The graphs showing the evolution of the Fitness Function, the CM component and the evolution of the VEE component for the tested scenarios. From left to right: scenario #1, scenario #2, scenario #3, scenario #4. The numbers on the curves represent the GB particle at each iteration: GB^p.](image)
The meter deployment location for each scenario is shown in TABLE 5. As expected, scenarios considering pseudo measurements (e.g. scenario #3 and #4) require additional nodes to be monitored than scenarios considering SM data (e.g. scenario #1 and #2). This can be explained by the fact that the pseudo measurement data considers a larger standard deviation ($\sigma$) of the measurements than the SM data does. Thus, to fulfill observability requirements additional sensors are required.

In scenario #2 the meter deployment cost is heavily penalized in comparison to the estimation error. Therefore, no device is deployed and a poor state estimation is achieved.

TABLE 5. OPTIMAL METER LOCATION FOR THE SIMULATED SCENARIOS.

<table>
<thead>
<tr>
<th>Scenario#</th>
<th>V sensor Qty. nodes</th>
<th>I sensor Qty. branches</th>
<th>SM/ Pseudo</th>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>SM</td>
<td>0.23 0.57</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>SM</td>
<td>0.08 1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>Pseudo</td>
<td>0.25 0.39</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>Pseudo</td>
<td>0.09 0.56</td>
</tr>
</tbody>
</table>

IV. CONCLUSIONS AND FUTURE WORK

In this paper a method to optimally place meters in active LV distribution networks is presented. The method formulates a multi-objective optimization problem, which minimizes the low voltage state estimation error and the cost associated to a particular meter deployment configuration. A Binary Particle Swarm Optimization (BPSO) based approach is used to solve the optimization problem. The results provide a Pareto front showing all the equally optimal solutions, which can be used in the meter placement decision making process. The method was successfully applied on a modified Cigré LV benchmark network. The method can be applied to situations where smart meter measurements are available and situations where these measurements are not available. As expected, simulation results show an increment in the cost parameter and a decrement in the error estimation parameter when the number of deployed measurements is incremented.

The main contributions of this project are the problem formulation as a multi-objective optimization problem and the use of smart meters and pseudo-meters for system observability.

Future work will capture the unbalance behavior of the distribution lines as well as meshed topologies.

ACKNOWLEDGMENT

The authors would like to thank EU FP7 DISCERN project partners for facilitating cost and measurement uncertainty-related data. (http://www.discern.eu/). As well as Niels Blauwbroek for his input on the state estimation implementation phase.

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